Phase Analysis and Focusing of Synchrotron Radiation

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Abstract

High accuracy calculations of synchrotron radiation (SR) emitted by a relativistic electron show that the phase of the frequency domain electric field of SR differs from the phase of radiation of a virtual point source. These differences may result in the reduction of focusing efficiency of diffraction-limited SR, if the focusing is performed by conventional optical components optimised for point sources. We show that by applying a phase correction locally, one may transform the phase of SR electric field at a desired polarisation to that of a point source. Such corrections are computed for undulator radiation (planar and helical) and bending magnet radiation (central part and edges). The focusing of the corrected SR wavefront can result in the increase of peak intensity in the focused spot up to several times compared to the focusing without correction. For non-diffraction-limited radiation, the effect of the phase corrections is reduced. Due to this reason, the use of the proposed phase corrections in existing electron storage rings is essentially of interest in the photon energy range from infrared to VUV. All numerical calculations discussed in the paper were performed by means of the computer code SRW[2].

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[2] The "Synchrotron Radiation Workshop" (SRW) computer code is freely available from the ESRF web site: http://www.esrf.fr/machine/support/ids/Public/index.html, link "Software".

1. Introduction

Most optical components currently used with synchrotron radiation (grazing incidence optics, multi-layer mirrors, zone plates, etc.) are developed taking into account the spectral-angular distributions of synchrotron radiation. Yet it is almost always assumed that the phase of the SR electric field is identical to that of a "point source" located somewhere in the middle of an undulator or at the tangent point in a bending magnet. In reality, the synchrotron radiation is generated as relativistic electrons move along a macroscopic path in a magnetic field, and the wavefront produced is more complicated than that of a point source.

The questions related to the SR wavefront propagation were considered in a number of papers on different levels of detail. An extended study of SR focusing was presented in the pioneering paper [1] where the analytical methods of Gaussian optics were applied to estimate spatial resolution at electron beam imaging with synchrotron light. A general approach to treatment of SR propagation based on Wigner distributions was suggested in [2]. A particular case of focusing the bending magnet SR was considered in [3] taking into account the effect of truncation of the radiation by aperture. Practical optimisation of the resolution at electron beam size measurements using a bending magnet SR, with respect to the effects of aperture and depth of field, was performed in [4]. A method of computation of the focused bending magnet radiation treating the SR emission and diffraction on the aperture fully in the frame of the scalar diffraction theory was suggested in [5]. A few wave-optics computer codes allowing simulation of the SR wavefront propagation through beamline components have appeared recently [6, 7].

In this paper, we consider phase difference between the frequency domain SR electric field emitted by relativistic electrons in realistic experimental conditions in a storage ring, and that of a point source. The difference is characterised by the phase correction, which, after being added to the phase of the SR electric field, gives the phase of the point source. To estimate the effect of applying the phase correction to synchrotron radiation, we consider two options of a simple 1:1 imaging scheme. In one option, the SR is focused only by an aberration-free optical component simulated by a thin lens; in the other option, a numerically computed phase correction is applied to the SR wavefront just in front of the lens. Spectral flux per unit surface (called intensity throughout the rest of this paper) of the synchrotron radiation propagated to the image plane is compared for these two options.

The initial SR electric field is computed using a high accuracy near-field method. This computation preserves all the phase terms that are necessary for further propagation of the radiation through optical components in a beamline [7, 8]. The propagation was performed using the Fourier optics principles and the asymptotic expansion methods.

This paper treats emission and propagation of the single electron SR. Common sense and actual practice show that if SR source is not diffraction-limited, the key SR characteristics, such as peak intensity, focused spot size, photon energy bandwidth after monochromator, etc., in the best case of a high-quality optics mostly depend on electron beam emittance. The emittance masks many of the effects that one would observe with a “filament” electron beam. However, as the emittance is reduced in new generations of SR sources, the radiation becomes diffraction-limited for higher photon energies, and the effects related to the single electron emission become more significant.

The next section describes how the phase correction is computed. Sections 3 – 5 present computation results of emission and propagation of various types of synchrotron radiation, analytical approximations for the corresponding phase corrections and conditions on electron beam size and angular divergence for which the effects of the phase corrections are not flattened by non-zero electron beam emittance.

2. Formal Method to Compute Phase Correction

Let $E_{in}(x,z)$ be a scalar complex function of horizontal and vertical coordinates *x, z* describing one polarisation component (linear or circular) of the frequency domain electric field of radiation at wavelength λ in a transverse plane after which optical component(s) can be placed. The distance from the source (for example, from the middle of undulator) to that plane is approximately equal to *R*. We would like to apply a phase correction $\Phi_{cor}(x,z)$, i.e., a transformation

$$T(x,z) = \exp[i\Phi_{cor}(x,z)] \tag{1}$$

to the electric field $E_{in}(x,z)$, so that its phase would become identical to that of a point source:

$$E_{out}(x,z) = T(x,z)E_{in}(x,z) = A(x,z)\exp[i\pi\,(x^2+z^2)/(\lambda R)], \tag{2}$$

where $A(x,z)$ is the amplitude of the electric field (that we don't intend to modify by the transformation $T(x,z)$). Then immediately from eqs. (1) and (2) we obtain:

$$\Phi_{cor}(x,z) = \arg[\exp[i\pi\,(x^2+z^2)/(\lambda R) + i\Phi_0]/E_{in}(x,z)], \tag{3}$$

where, without loss of generality, the principal value of the argument function is assumed; Φ_0 is arbitrary constant. Eqs. (2) and (3) imply small observation angles (a condition that is, in practice, well satisfied in storage rings). It is also assumed that the amplitude of the electric field is never exactly equal to zero within the range where the transformation $T(x,z)$ is applied.

To make practical use of eq. (3), one needs to know the electric field $E_{in}(x,z)$. In the case of synchrotron radiation, the electric field emitted by relativistic electrons moving in arbitrary magnetic field can be computed to a very high degree of accuracy [7, 8]. If the electric field is known numerically, we obtain a numerical representation of the phase correction via eq. (3). In a number of cases, analytical approximations for the phase corrections can be obtained. Such approximations are given in the following sections for undulator and bending magnet radiation.

We have chosen to transform the phase of the SR electric field to that of a point source, because after such transformation, one can safely apply standard optical components dedicated to point sources for further propagation of the radiation in a beamline. One can also apply the above formalism to deduce a more favourable transformation for particular conditions. For example, if one sets *R* = -*R'* in eq. (3), one obtains a phase shift introduced by an optical component which both corrects the phase of the input radiation and makes focusing of this radiation at the distance *R'* downstream of the optical component.

It is easy to show that the use of the phase correction (3) with a focusing optical component dedicated to a point source results in the highest peak intensity (at a given polarisation) in the image plane, which is possible to obtain without modifying the amplitude of the electric field at the optical component. However, the correction (3) may not guarantee the smallest possible spot size in the image plane. These two requirements (highest peak intensity and smallest spot size) are not contradictory in most cases. However, there are cases when, after applying the phase correction (3) at the focusing, the intensity distribution in the image plane is still not identical to that obtained when focusing a uniform wavefront from a point source (for example, there can be relatively large secondary maximums in the focused spot).

It is important to note that the phase correction (3), which is derived for only one polarisation component of the SR electric field, does not guarantee a proper transformation for another polarisation component of the electric field. In many practical cases, however, only one SR polarisation component is dominating (linear horizontal in planar undulators, circular right or left in helical undulators, etc.), so this obstacle may not pose a big problem.

This paper does not treat in detail the question of possible practical implementation of the phase corrections to the SR. Nevertheless we would like to mention that for normal incidence transmission optics, the phase shift is proportional to the thickness of the material of the optical component. This effect is used for the manufacture of zone plates and refractive lenses for X-rays [9]. Therefore, one of the most straightforward implementations of the phase correction derived by eq. (3) could be a kind of special zone plate.

3. Undulator Radiation

Let us consider the radiation emitted by a filament electron beam at an energy of 6 GeV, current 0.2 A in a planar undulator with 38 periods of 42 mm, peak magnetic field of 0.56 T (U42 undulator at the ESRF). The emitted radiation is initially considered at a distance of 30 m from the undulator, where a focusing component should be placed. The goal is to obtain the highest peak intensity and the smallest spot size at 30 m from the optical component (1:1 imaging).

As the first example related to focusing of undulator radiation (UR), we computed intensity distribution of the UR focused by a thin lens, for several photon energies close to the on-axis peak value of the fundamental harmonic. The computation results are presented in figure 1. As one can see, the maximum peak intensity is obtained at the photon energy of 2.36 keV, which is slightly lower than the on-axis peak value of the fundamental. For this photon energy, the intensity distribution at the lens represents a small ring, rather than a cone with a maximum on axis (left graph in figure 1). This feature of undulator radiation was discussed in [7].

The phase correction was then computed for the UR at a photon energy of 2.36 keV, which gave the highest peak intensity in the focused spot, and the focusing of this radiation with the phase correction applied at the longitudinal position of the lens was simulated. The intensity distribution at the lens, the phase correction according to eq. (3), and the intensity in the plane of 1:1 imaging of the UR focused with and without the phase correction are shown in figure 2. As one can see, the use of the phase correction allows the peak intensity in the focused spot to increase further (by a factor of ~ 1.8 in the example considered).

The phase correction for odd harmonics of radiation from a planar undulator mainly consists in changing the phase of the rings surrounding the central cone (or the main central ring), consecutively by π. Here we consider the rings immediately following the central cone (or the central ring) and belonging to the same harmonic. If the aperture of an optical component can only accept a part of the UR central cone, there is no sense in applying any phase correction.

In the next example we consider the second harmonic of the UR from the same undulator at the same observation conditions. Even harmonics of radiation from a planar undulator are well known to be suppressed in the on-axis direction, however, a considerable photon flux is emitted into the rings with diameters and widths dependent on the photon energy value. Figure 3 presents computation results of the UR intensity at 4.775 keV photon energy (second harmonic) at the lens, the phase correction, and intensity distribution in the plane of 1:1 imaging, if focused with and without the phase correction.

The phase correction for even harmonics of radiation from a planar undulator consists in the same type of correction as for odd harmonics and, in addition to this, in changing the phase by π at $x > 0$ compared to $x < 0$. The latter is a consequence of anti-symmetry of the horizontal electric field over the horizontal position at even harmonics. We note that a similar situation takes place for the vertical polarisation component of conventional bending magnet radiation, which is anti-symmetrical over the vertical position. Due to this anti-symmetry, there is no peak intensity in the centre of the focused spot in these cases. As we see from figure 3, the use of a simple half-wave shift at $x > 0$ (or at $x < 0$) removes the anti-symmetry of the electric field and thus brings the maximum intensity to the centre of the focused spot. However, the use of the full phase correction shown in the middle of figure 3, results in considerably higher peak intensity in the spot (right graph in figure 3).

As the last illustration of phase corrections for UR, we consider the case of a helical undulator. The phase correction for a fundamental harmonic of radiation from a helical undulator looks identical to that of odd harmonics of a planar undulator (figure 2), however, the situation with the other harmonics is different. The harmonics higher than one are not emitted in the on-axis direction in helical undulators. Due to this obstacle, very often they are not even considered for practical use. Yet there is a lot of circularly polarised radiation that is emitted into rings at these harmonics. This radiation could be well used in experiments.

To illustrate, we take the following parameters: filament electron beam at an energy of 6 GeV, current 0.2 A in a helical undulator with 28 periods of 52 mm, peak horizontal and vertical magnetic fields of 0.3 T. The radiation at 4.20 keV photon energy (second harmonic) is initially considered at a distance of 30 m from the undulator, where a focusing optical component should be placed. As in previous examples, the goal is to obtain the highest peak intensity and the smallest spot size at 30 m from the optical component. Figures 4 and 5 show the results of simulations.

The phase correction for higher harmonics of a helical undulator consists in the same type of correction as for odd harmonics of a planar undulator (see figure 2) and, in addition to this, in the linear change of the phase with an azimuth angle (figure 4). The effect of the phase correction is greater in this case compared to other UR cases considered (the peak intensity increased by a factor of 8 in our example).

The phase correction for different types of undulator radiation at harmonic number *n* (central cone and adjacent rings) can be approximated as follows:

$$\begin{aligned}
&\Phi_{cor\,UR} \approx \pi\eta(\theta-\theta_{nN-l_0}) + \pi g(n,\theta_x,\theta_z) + \pi\,\mathrm{sgn}(\theta_{nN-l_0}-\theta)\sum_{k=0}^{k_{\max}}\eta(\theta-\theta_{2k})\eta(\theta_{2k+1}-\theta),\\
&\theta_l = (2\lambda/L)^{1/2}\left| l+l_0-\lambda_1 N/\lambda \right|^{1/2}\eta(l),\\
&l_0 = [\lambda_1 N/\lambda]+1,
\end{aligned} \tag{4}$$

where η is the step function, sgn is the sign function:

$$\eta(x) \equiv \begin{cases} 0, & x<0, \\ 1, & x\ge 0, \end{cases} \quad \mathrm{sgn}(x) \equiv \begin{cases} -1, & x<0, \\ 1, & x\ge 0. \end{cases}$$

θ_x, θ_z are horizontal and vertical observation angles ($|\theta_x| << 1, |\theta_z| << 1$), $\theta \equiv (\theta_x^2 + \theta_z^2)^{1/2}$; N is the number of periods, λ_1 the wavelength of the fundamental harmonic on axis. k_{max} is obtained from the relation: $\theta_{2k_{max}} \le \theta_{max}$, with θ_{max} giving the angular aperture. Square braces in the expression for l_0 mean an integer part of a number (in other parts of this paper, the square braces do not have such meaning). Function $g(n, \theta_x, \theta_z)$ is specific to the type of undulator. For a planar undulator:

$$g(n, \theta_x, \theta_z) \approx \begin{cases} 0, & n = 1,3,... \\ \eta(\theta_x), & n = 2,4,... \end{cases} \quad (5)$$

for a helical undulator:

$$g(n, \theta_x, \theta_z) \approx (-1)^p (n-1) \left[\arccos\left[[2\eta(\theta_z) - 1]\theta_x / \theta \right] / \pi - \eta(\theta_z) + 1 \right], \quad (6)$$

where arccos and step functions are used to express the azimuth angle via θ_x, θ_z; $p = 0$ or 1 depending on the UR polarisation (right or left). The approximation (6) is valid only at $\lambda > \lambda_1 / n$, with improved accuracy for larger wavelengths.

The approximations (4) – (6) accord with the phase corrections calculated numerically using the near-field SR computation method (figures 2 - 4). We note that these corrections do not depend on the observation distance. The reason for this is that regarding the above consideration, we limited ourselves to relatively small angles comparable with the UR central cone or small rings of emission at given harmonics. The dependence on the observation distance may appear as one considers larger observation angles. In such cases the formulae (4) – (6) may not be valid. However, one can still treat the problem numerically (based on the eq. (3) or a more sophisticated algorithm). We emphasise the necessity of near-field SR computation for such purposes. The near-field effects in the off-axis undulator radiation were discussed in [10].

For efficient practical use of the phase corrections for undulator radiation, the apparent angular divergence of the electron beam should be smaller than the angular width of the largest UR ring accepted by the aperture of the optical component, and the transverse size of the electron beam should be at least smaller than the spot size of the focused UR central cone (assuming 1:1 imaging). As a very rough estimation,

$$(\Delta'^2_{x,z} + \Delta^2_{x,z}/R^2)^{1/2} << (\lambda/L)^{1/2},$$
$$\Delta_{x,z} << (\lambda L)^{1/2}, \quad (7)$$

where $\Delta'_{x,z}, \Delta_{x,z}$ mean horizontal (vertical) angular divergence and transverse size of the electron beam respectively, *L* is the lens of the undulator. In the case of even harmonics of a planar undulator, or harmonics $n > 1$ of a helical undulator, one can have some gain even if the condition on the angular divergence (first relation in (7)) is not satisfied.

It should be pointed out that it formally follows from (7) that the source should be diffraction-limited: $\Delta_{x,z}\Delta'_{x,z} << \lambda$. In third-generation SR sources, the conditions (7) can be met in the soft X-ray or VUV range, at least for the vertical divergence and size of the electron beam.

4. Conventional Bending Magnet Radiation

Consider a case of bending magnet radiation at the following emission and observation conditions: electron beam energy of 2.5 GeV, current 0.5 A; constant field in bending magnet 1.56 T; photon energy 40 eV, horizontal angular aperture 6 mr, vertical aperture 4 mr (~natural opening angle of the bending magnet SR); distance from tangential source point to optical component 5 m.

Surface plots of intensity distributions and phase corrections for the horizontal and vertical SR polarisation components computed numerically at these conditions are shown in figure 6. The branches of the phase function equal to zero at zero transverse positions were taken in eq. (3). The phase correction for the vertical component of the electric field differs from the correction for the horizontal component by a π- flip at zero vertical position. This is a consequence of anti-symmetry of the vertical electric field with respect to the plane of the electron beam orbit.

Computation results of intensity in the plane of 1:1 imaging are presented in figure 7. Due to the phase differences of the bending magnet SR from that of a point source, there is an aberration in the focused spot, if the focusing is performed by an optical component dedicated to a point source (left plots in figure 7). For single electron radiation, this aberration appears when the horizontal aperture is comparable with the natural opening angle of the bending magnet SR, and it becomes larger as the horizontal aperture further increases. These results accord with the simulations performed earlier by another computation method that gives the intensity distribution of the focused SR in the image plane without the need to compute the SR wavefront at the position of the lens [5, 11].

Middle and right plots in figure 7 show the effects of the phase corrections for the horizontal and vertical components of the electric field. The phase correction for the horizontal electric field (upper right plot in figure 6) removes large secondary maximums of intensity distribution of the focused SR, reduces the RMS spot size and increases the peak intensity (upper right plot in figure 7). The peak intensity is increased by more than a factor of 2 in the example considered. The effect becomes more valuable with the increase of the horizontal aperture.

The phase correction for the vertical electric field (lower right plot in figure 6) brings the main maximum of the focused vertically polarised SR to the centre of the spot (lower middle plot in figure 7) by compensating the anti-symmetry of the vertical electric field. Yet this correction does not seem very practical, because it introduces anti-symmetry to the horizontal electric field and thus splits the main maximum of the focused horizontally polarised SR. The vertical polarisation component of the visible focused bending magnet SR was used for the electron beam size measurements in [11], where the presence of a vertically split main maximum in the focused spot allowed a more precise estimation of a small vertical size of the electron beam, than with the horizontal SR polarisation component.

The phase correction for the conventional bending magnet SR can be represented by the following expressions:

$$\begin{aligned}\Phi_{cor\ BM\ hor} &\approx -(\pi/\lambda)\rho\theta_x(\gamma^{-2}+\theta_x^2/3+\theta_z^2),\\ \Phi_{cor\ BM\ vert} &= \Phi_{cor\ BM\ hor}+\pi\eta(\theta_z),\end{aligned} \tag{8}$$

where λ is the radiation wavelength, γ reduced electron energy, ρ bending radius, θ_x and θ_z observation angles. Eqs. (8) imply small observation angles ($|\theta_x| << 1, |\theta_z| << 1$) and a small longitudinal size of the emission region compared to the observation distance ($\min[(\lambda\rho^2)^{1/3}, \lambda\gamma^2] << R$). Both requirements are well satisfied in storage rings. The agreement of eq. (8) with numerical computation (figure 6) is of the order of one percent at the edges of the aperture (the computation assumes a negative bending radius for a positive magnetic field). Note that eq. (8) does not depend on the observation distance, i.e. the phase difference between the bending magnet SR and a point source is not a consequence of near-field effects:

it can take place at large distances from the bending magnet, if the horizontal aperture is large enough.[3]

The calculations illustrated by figures 6 and 7 were done for a filament electron beam. For the effects described to take place with a finite-emittance beam, one should ensure, firstly, that the apparent angular divergence of the electron beam is considerably smaller than the natural opening angle of the bending magnet SR, and secondly, that the transverse size of the electron beam is smaller than the spot size of the focused single electron SR (assuming 1:1 imaging):

$$\begin{aligned} &(\Delta'^2_{x,z} + \Delta^2_{x,z} / R^2)^{1/2} << \min[\lambda\gamma^2 / |\rho|, (\lambda / |\rho|)^{1/3}], \\ &\Delta_{x,z} << \max[|\rho|\gamma^{-2}, (\lambda^2 |\rho|)^{1/3}]. \end{aligned} \qquad (9)$$

The arguments of the min function are the estimations of the SR opening angle for wavelengths larger and smaller than the critical wavelength for the bending magnet. In practice, the first relation in (9) is met in synchrotron radiation sources up to very hard X-rays; however, the second one can be satisfied for typical values of the horizontal size of the electron beam only in the infrared range.

5. Edge Radiation

High brilliance of long-wavelength radiation generated by relativistic electrons at bending magnet edges in storage rings (edge radiation, ER) was pointed out in [13-16]. Experimental results with the infrared range ER were reported in [17, 18]. Practical application to the visible range ER for electron beam emittance diagnostics was described in [19, 20].

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[3] When this paper was prepared for publication, the authors were shown that the phase term analogous to eq. (8) was considered in [12].

To illustrate numerically the emission and focusing features of the infrared ER, we take the following set of parameters: electron beam energy of 2.5 GeV, current 0.5 A; constant field in bending magnet of 1.56 T, straight section length of 7 m; radiation wavelength of 10 μm. The intensity distribution of the ER in the transverse plane located at 5 m from the downstream bending magnet edge is shown in figure 8. The ring pattern is a result of interference of the light emitted at two bending magnet edges bounding the straight section, and a near-field effect [15]. The numerically computed phase correction that makes the phase of the horizontal component of the ER electric field identical to that of a point source located at the downstream bending magnet edge, is shown in figure 9 (left) as an image plot. At that computation, the principal value of the arg function between -π and π was taken in eq. (3).

The phase correction for the horizontal polarisation component of the near-field edge radiation (i.e. at a distance from the downstream magnet edge $R < \lambda\gamma^2$) for transverse positions close to the straight section axis ($r^2 \equiv x^2 + z^2 \sim \lambda R$) can be approximated as:

$$\Phi_{cor\ ER} \approx \pi r^2 L / [2\lambda R(R+L)] + \pi\eta(x) + \pi \sum_{k=1}^{k_{\max}} \eta(r - r_{2k-1})\eta(r_{2k} - r), \qquad (10)$$
$$r_l = [2\lambda R(R+L)\, l/L]^{1/2},$$

where L is now the straight section length, r_l are the radii of black rings (minima of the ER intensity distribution, see figure 8); $k_{\max}$ is obtained from the relation: $r_{2k_{\max}} \le r_{\max}$ with $r_{\max}$ giving the aperture of the optical component, $\eta(x)$ is the step function. The correction (10) transforms the phase of the ER (horizontal polarisation component) to that of a virtual point source at the downstream magnet edge.

At larger apertures, the phase shift of the optical component dedicated to focusing the horizontal polarisation component of the ER from two bending magnet edges limiting one straight section can be roughly represented as:

$$\Phi_{foc\,ER} = \Phi_{Thin\,Lens} + \Phi_{cor\,ER} \approx -(\pi/\lambda)(x^2 + z^2)\big/ f(x),$$
$$f(x) = \begin{cases} RR'/(R+R'), \ x<0 \\ (R+L)R'/(R+L+R'), \ x \ge 0 \end{cases} \qquad (11)$$

where *R'* is the distance from the optical component to the image plane. It is assumed that the downstream magnet bends the electron beam towards x < 0. The phase shift (11) is shown in figure 9 (right) as surface plot. An advantage of using an optical component with the phase shift (11) is that such a component, if realised for example as a composed focusing mirror, would not depend on wavelength.

Figure 10 presents computation results of focusing the edge radiation with several options of optical component. In each option, the optical component is located at *R* = 5 m from the downstream bending magnet edge and the goal is to obtain high peak intensity and small spot size in the plane at *R'* = 5 m from the optical component. The aperture of the optical component is 10 cm x 10 cm. Figure 10 shows that if one focuses the ER with an optical component dedicated to a point source, then a high peak intensity and a relatively small spot size in the observation plane is attained when the focal distance of the optical component is approximately equal to $RR'/(R+R')$ or $(R+L)R'/(R+L+R')$, i.e. when the source is assumed to be close to the downstream or the upstream magnet edge. However, in these two cases the focused spot is still not as small as it would be for a point source. The situation improves if the optical component with a phase shift given by eq. (11) is applied. We see that the peak intensity in this case is close to the one obtained with the phase correction computed numerically by eq.(3).

It is important to note that for the phase shift represented by eq. (11) to give results, the aperture of the optical component should be large enough, at least larger than $(\lambda R)^{1/2}$ in the near-field observation conditions. For very large apertures, one can consider adding to eq. (11) also a type of phase correction dedicated to conventional bending magnet SR (see section 4). If an extra limiting aperture is located between the downstream bending magnet edge and the focusing component, it may happen to block a large portion of radiation from the upstream magnet edge and modify the wavefront by diffraction, so the use of the optical component with the phase shift (11) may appear inefficient. However, in such a case one can still consider the phase correction of the radiation according to eq. (3). If necessary, one can use in eq. (3) the electric field computed with regard to the diffraction on limiting aperture(s) before the optical component. The Fourier-optics methods implemented in the SRW code allow the efficient computation of the wavefront propagation through apertures and drift spaces.

For the phase correction (10) to give practical results, the electron beam angular divergence and transverse size should be small enough, at least to satisfy that

$$\begin{aligned} &(\Delta'^2_{x,z} + \Delta^2_{x,z}/R^2)^{1/2} \ll [\lambda R(R+L)/L]^{1/2}, \\ &\Delta_{x,z} \ll [\lambda RL/(R+L)]^{1/2}. \end{aligned} \qquad (12)$$

Both conditions are met for the infrared range ER in electron storage rings. In the case of the phase shift (11) the first requirement in (12) can be omitted.

4. Summary

The phase of the frequency domain electric field of synchrotron radiation emitted by a relativistic electron in magnetic fields of a storage ring differs from that of a point source.

Some of the differences can be observed not only at small distances from the emission region, yet also in the far-field region.

In most practical cases met in SR sources, the phase corrections converting the phase of diffraction-limited monochromatic SR to that of the radiation of equivalent point source can be precisely computed. The intensity distributions of the radiation focused with and without the phase corrections can also be computed to a high accuracy using the wavefront propagation methods based on Fourier optics and asymptotic expansions. In the case of focusing the diffraction-limited SR, the use of the phase corrections can result in peak intensity up to several times higher in the focused spot. The effect depends on particular SR emission and propagation conditions.

For a non-diffraction-limited SR, the effect of the phase corrections is reduced. For this reason, the use of this technique in existing SR sources seems to be limited to the photon energy range from IR to VUV. The method of the phase corrections could be considered for the next generations of SR sources and free-electron lasers.

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Figure Captions

Figure 1. Intensity distribution of UR (horizontal cuts by the median plane) for four photon energies close to the on-axis peak value of the fundamental harmonic, at the position of the lens 30 m from the undulator (left) and in the plane of 1:1 imaging (right). The lens aperture is 3.4 mm x 3.4 mm.

Figure 2. Left: intensity distribution (horizontal cut by the median plane) of horizontally polarised radiation emitted at the fundamental harmonic of a planar undulator at 2.36 keV photon energy (which is slightly smaller than the on-axis peak value of the fundamental, and gives the highest peak intensity in the spot if focused by a perfect lens), at the longitudinal position of the lens. Middle: phase correction computed numerically. Right: intensity distributions in the image plane (horizontal cuts) for the UR focused without any phase correction (a), and with the phase correction applied at the longitudinal position of the lens (b).

Figure 3. Left: intensity distribution (horizontal cut by the median plane) of horizontally polarised radiation emitted at the second harmonic of a planar undulator (4.775 keV photon energy) at the longitudinal position of the lens. Middle: phase correction computed numerically. Right: intensity distributions in the image plane (horizontal cuts) for the UR focused without any phase correction (a), with half-wave correction at $x > 0$ (b), and with the full phase correction applied at the longitudinal position of the lens (c). The aperture at the lens is 2.4 mm x 2.4 mm.

Figure 4. Left: intensity distribution (horizontal cut by the median plane) of circularly polarised radiation emitted at the second harmonic of a helical undulator (4.20 keV photon

energy) at the longitudinal position of the lens. Middle: phase correction computed numerically. Right: intensity distributions in the image plane (horizontal cuts) for the UR focused without any phase correction (a), and with the phase correction applied at the longitudinal position of the lens (b). The aperture at the lens is 2.4 mm x 2.4 mm.

Figure 5. Intensity distribution of circularly polarised radiation emitted at second harmonic of a helical undulator (4.20 keV photon energy) in the plane of a focusing component 30 m from the undulator (left), and in the image plane, if focused without (middle) and with (right) the phase correction. The grey scale is normalised by the maximum intensity separately in each image plot.

Figure 6. Left: intensity distribution of the bending magnet radiation in transverse plane at 5 m from the tangential “source point”. Right: phase corrections. Upper plots correspond to the horizontal, lower plots to the vertical polarisation component of the bending magnet SR.

Figure 7. Intensity distributions of the focused bending magnet SR. Upper plots correspond to the horizontal, lower plots to vertical SR polarisation component. Left: focusing by aberration-free “thin lens”, without any phase correction. Middle: focusing with the phase correction applied at the longitudinal position of the lens (the correction is different for horizontal and vertical polarisation components). Right: horizontal (for horizontal polarisation component) and vertical (for vertical polarisation component) cuts of intensity in the spot for the focusing without (a) and with (b) the phase correction.

Figure 8. Intensity distribution of infrared edge radiation at 10 μm wavelength as observed in transverse plane 5 m from the downstream bending magnet edge. Left: image plot. Right: horizontal cut by the median plane.

Figure 9. Left: phase correction that makes the phase of horizontal component of the ER electric field identical to that of a point source located at the downstream magnet edge (bending the electron beam towards $x < 0$). Middle: phase shift introduced by aberration-free “thin lens” assuming point source at the downstream magnet edge. Right: phase shift of the optical component composed of two “thin lenses” with different focal distances at $x < 0$ and $x > 0$. The focal distances correspond to the location of point sources at the downstream and upstream magnet edges.

Figure 10. Horizontal intensity profiles of the edge radiation in the image plane for the focusing by different options of optical component.

a: thin lens with a focal distance of $f = (R + L/2)R'/(R + L/2 + R')$, i.e. assuming a point source in the middle of the straight section;

b: thin lens with a focal distance of $f = RR'/(R + R')$, i.e. assuming a point source at the downstream magnet edge;

c: thin lens with a focal distance of $f = (R + L)R'/(R + L + R')$, i.e. assuming a point source at the upstream magnet edge;

d: optical component with the phase shift represented by eq. (8);

e: thin lens and the phase correction computed by eq. (3).

In each of the options, the optical component is located at R = 5 m from the downstream magnet edge and the image plane is at R' = 5 m from the optical component.